THEORY OF PSYCHROMETRY

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UDC 551.508.71

A consistent theory of psychrometry is presented.

A century and a half has passed since the appearance of the first work [1, 2] on the problem of psychrometry. Nevertheless, the problem cannot be considered solved at the present work from either the theoretical or practical aspect.

The relationship widely accepted in meteorology contains an empirically determined quantity – the psychrometer coefficient. Under the generally accepted theory based on the concept of adiabatic saturation, no one has managed to obtain the correct value of this coefficient or even to explain the qualitative nature of its variation as a function of various factors.

We have made an attempt to construct a theory for the psychrometer which would make it possible to obtain and explain presently known experimental results and which would not require the introduction of any additional information of an empirical nature.

We consider a plane surface on which a phase transition occurs (evaporation or sublimation of material). At some distance δ from it, let there be another surface (removal surface) at which fixed values are maintained for the temperature and partial pressure of the evaporating material. Then the temperature of the phase transition takes on a strictly determined value during the period of time needed for establishment of equilibrium. We determine this temperature under conditions where the heat required for maintenance of a stable process is supplied to the phase-transition surface by means of the thermal conductivity of the ambient gas mixture and by radiation between the surfaces.

Under assumptions previously stated [3, 4], the problem is formulated in the following manner:

basic equations:

$$\lambda \frac{d^2T}{dx^2} - c_p\rho v \frac{dT}{dx} = 0, \tag{1}$$

$$\frac{d}{dx}(\rho v) = 0, \tag{2}$$

$$\frac{d}{dx}(\rho v c) + \frac{d}{dx}(j) = 0, \tag{3}$$

$$\rho = \frac{P}{RT} \frac{1}{\left(\frac{c}{M_v} + \frac{1-c}{M_g}\right)}, \tag{4}$$

boundary conditions at x = 0,

$$T = T_1, \tag{5}$$

$$c = c_1, \tag{6}$$

Moscow Institute of Chemical Machinery Construction. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 28, No. 4, pp. 698-704, April, 1975. Original article submitted December 30, 1973.

at x = L

$$\lambda \frac{dT}{dx} + \varepsilon\sigma (T_1^4 - T_0^4) = \mu H, \tag{7}$$

$$v\rho = -v_{bo}\rho_{sol}\left(1 - \frac{\rho}{\rho_{sol}}\right), \tag{8}$$

$$v\rho c + j = \mu. \tag{9}$$

Omitting the intermediate calculations, we present the final results,

$$\frac{\lambda}{c_p} \ln\left(1 + \frac{T_1 - T_0}{H\psi}\right) = \frac{PD \ln\left(\frac{P - P_1}{P - P_0}\right)}{R_c\left\{T_0 + \frac{T_1 - T_0}{\ln\left(1 + \frac{T_1 - T_0}{H\psi} c_p\right)} - \frac{H}{c_p}\psi\right\}}, \tag{10}$$

$$\mu = \frac{\lambda}{c_p\delta} \ln\left\{1 + \frac{T_1 - T_0}{H\psi} c_p\right\}, \tag{11}$$

where

$$\psi = 1 - \frac{\varepsilon\sigma (T_1^4 - T_0^4)}{\mu H}. \tag{12}$$

Equations (10) and (11) are very cumbersome and unsuitable for application. However, if one limits the applicability of the solution to conditions characteristic of psychrometer operation, considerable simplification is possible. In fact, for these conditions

$$\frac{(T_1 - T_0) c_p}{H\psi} \ll 1 \tag{13}$$

and therefore the approximation ln (1 + x) ≈ x is completely acceptable on the left side of Eq. (10) and on the right side of Eq. (11). However, as far as the expression in the denominator on the right side of Eq. (10),

$$T_0 + \frac{T_1 - T_0}{\ln\left(1 + \frac{T_1 - T_0}{H\psi} c_p\right)} - \frac{H}{c_p}\psi,$$

is concerned, a simple evaluation shows that the assumed approximation is not satisfactory and therefore it becomes necessary to keep two terms in the expansion for the logarithm:

$$\frac{\lambda}{H} (T_1 - T_0) + \frac{\delta\varepsilon\sigma (T_1^4 - T_0^4)}{H} = \frac{2PD}{R_1 (T_1 + T_0)} \ln\left(\frac{P - P_1}{P - P_0}\right), \tag{14}$$

$$\mu = \frac{\lambda}{\delta} \frac{T_1 - T_0}{H} + \frac{\varepsilon\sigma (T_1^4 - T_0^4)}{H}. \tag{15}$$

Note that the error introduced by these simplifications is practically always less than 1%.

In many cases which are realistic for practically all meteorological measurements of humidity, $P_1 < P_0 \ll P$, and consequently we have with sufficient accuracy

$$\ln \frac{P - P_1}{P - P_0} = \frac{P_0 - P_1}{P} \tag{16}$$

and

$$\frac{\lambda}{H} (T_1 - T_0) + \frac{\delta\varepsilon\sigma (T_1^4 - T_0^4)}{H} = \frac{2P_d D}{R_1 (T_1 + T_0)} \frac{P_0 - P_1}{P}. \tag{17}$$

For evaporation or sublimation from the surfaces of a sphere [5] or a cylinder of infinite length, the problem is solved similarly. In these cases, the form of Eq. (17) remains as before but the following quantities must be substituted for δ:

for a sphere

$$\frac{r_0 (r_1 - r_0)}{r_1} \tag{18}$$

for a cylinder

$$\frac{1}{2\pi}\ln\left(\frac{r_1}{r_0}\right). \tag{19}$$

It is advisable to emphasize that Eq. (17) can be used directly only for calculations under conditions where the medium is static.

It is interesting to note that where the evaporation is from a spherical surface where r → ∞ (i.e., in a medium of unbounded size), Eq. (17) takes on a form corresponding to the expression obtained by Maxwell [5] while for a cylinder and plate with r → ∞ and $\delta \to \infty$, respectively (in a static medium of unbounded size), no psychrometric difference whatever should be observed as is clear from Eq. (17). What has been said indicates that the Maxwell theory of the psychrometer is only applicable for a spherical body (under the condition that special measures have been taken to eliminate free convection).

In practice, as a rule, the medium is displaced with respect to the phase-transition surface as the result of forced or free motion.

The use of Eq. (17) under these conditions immediately encounters the difficulty arising out of the uncertainty of the value of δ. The "film theory" presents certain possibilities here by which the actual, quite complex process of exchange devloping between the phase-transition surface and the moving medium can be reduced to transfer into some static film directly overlying that surface. In such a case, the film thickness is determined from the relation

$$\delta = l/\mathrm{Nu}. \tag{20}$$

The quite extensive volume of data accumulated up to now can be used for the determination of Nu.

In order to justify the admissibility of the proposed approach, we employ the results of experimental psychrometer studies known to us. Unfortunately, only a few of them can be introduced for a quantitative verification of the proposed theory; in most cases, the data lacks the completeness necessary for verification. Therefore such results can only be used for a qualitative check of the theory.

In this paper, we confine ourselves to the experimental data of Usol'tsev [6] for the purpose of quantitative verification; the data was obtained for a standard aspiration psychrometer (type M-34) during studies aimed at refinement of the value for the psychrometer coefficient.

The quantities needed for a calculation based on Eq. (17) are then determined in the following manner. Assuming that the initial section is blunt and the diameter of the bulb and covering is sufficiently large in comparison with the thickness of the boundary layer (which makes it possible to neglect transverse curvature), we use the following expression for Nu [7]:

$$\mathrm{Nu} = 0.0266\,\mathrm{Re}^{0.8}. \tag{21}$$

Next, using the usual relations for the calculations of radiative heat transfer, we obtain a value $\varepsilon = 0.38$ for the reduced emissivity (the emissivity for water, ice, and the internal surface of the cylinder surrounding the mercury bulb and covering was assumed to be 0.960, 0.925, and 0.041 respectively [8]). The coefficients λ and D are determined from the arithmetical mean temperature.

The experimental data and the theoretical results are given in Table 1. As is obvious, they are in particularly good agreement in the case of evaporation of water. As far as some of the discrepancies at negative temperatures are concerned, they can be explained most quickly by partially ice-free areas in the muslin, which is completely likely since movement of frozen moisture within the muslin is impossible. As far as the highly disparate data in the last column of the table is concerned, it should be considered the result of error.

The exceptionally high accuracy of the agreement between theoretical results and experimental data is a basis for considering the proposed theory sufficiently satisfactory.

We compare the data of the proposed theory with other experimental and theoretical results for further analysis of the possibilities of applying the proposed theory to the problems of psychrometry. We write the basic equations (17) in the form

$$P_1 = P_0 - \left[\frac{\lambda}{H}\,\frac{R_1(T_1+T_0)}{2P_dD} + \frac{R_1(T_1+T_0)^2\delta\varepsilon\sigma}{H2P_dD}(T_1^2+T_0^2)\right]P(T_1-T_0). \tag{22}$$

TABLE 1. Theoretical Calculations and the Observations of Usol'tsev

Experiment No.	Experimental observations				Calculated partial pressure of water vapor, P_1, mm Hg	Relative disagreement between calculated and observed values of P_1, %
	dry-bulb thermometer T_1, °C	wet-bulb thermometer T_0, °C	barometric pressure, P mm Hg	partial vapor pressure in chamber, P_1, mm Hg		
Water on thermometer bulb						
1	22,1	6,76	750	0,199	0,198	0,5
2	22,3	7,06	750	0,270	0,285	5,5
3	22,1	6,96	750	0,306	0,311	1,63
4	22,3	7,06	750	0,297	0,285	4,04
5	21,1	6,36	756	0,201	0,207	2,9
6	21,8	6,76	756	0,214	0,212	0,94
7	21,9	6,76	756	0,230	0,227	1,3
8	21,9	6,86	754,5	0,199	0,212	6,53
Ice on thermometer bulb						
1	—5,1	—9,43	748	0,256	0,276	7,81
2	—3,8	—8,43	747	0,354	0,356	0,565
3	—3,0	—8,14	747	0,212	0,218	2,83
4	—2,6	—7,43	747	0,447	0,462	3,24
5	—2,5	—7,13	746	0,514	0,562	9,33
6	—6,0	—10,03	736	0,288	0,294	2,08
7	—5,4	—9,33	736	0,437	0,454	3,89
8	—4,3	—8,68	734	0,329	0,411	24,9

The expression in the square brackets is none other than the quantity known in the literature as the psychrometer coefficient A,

$$A = \frac{\lambda}{H}\,\frac{R_1(T_1+T_0)}{2P_dD} + \frac{R_1(T_1+T_0)}{2P_dD}\,\frac{\varepsilon\sigma\delta(T_1^2+T_0^2)(T_1+T_0)}{H}. \tag{23}$$

This equation not only determines the numerical value of the coefficient A but also makes it possible to explain the role of the various factors. Before turning to an analysis of the separate factors, we express the transport coefficients λ and D as explicit functions of thermodynamic parameters,

$$\lambda = \lambda_0\left(\frac{T}{T_0}\right)^{n-1}, \tag{24}$$

$$P_dD = P_0D_0\left(\frac{T}{T_0}\right)^{n}; \tag{25}$$

we then have

$$A = \frac{\lambda_0}{H}\,\frac{R_1T_0}{P_0D_0} + \frac{R_1(T_1+T_0)}{2P_dD}\,\frac{\delta\varepsilon\sigma(T_1^2+T_0^2)(T_1-T_0)}{H}. \tag{26}$$

The expression on the right side of Eq. (26) consists of two terms. The first is a group made up of thermophysical characteristics and is slightly dependent on temperature (in proportion to the variation in H). The second term, which is really responsible for the variation in A, is determined by the radiation effect.

The analysis of this term should be discussed.

As is clear from Eq. (26), an important factor is the rate of ventilation. When it increases, Nu increases, δ decreases, and consequently the term becomes smaller; this is the effect of the structural design of the working portion of the wet-bulb thermometer. This effect appears both through radiative heat transfer and through the flow around the mercury bulb, which depends on the shape of the covering. For example, if in the experiment discussed above the covering was made in such a way as to ensure an uninterrupted flow of air at the bulb, the transfer conditions change, which entails a change in Nu and, consequently, in the value of δ; this is the effect of temperature conditions, a characteristic feature of which is that both temperature difference and the absolute value of the temperature are important.

We note that these conclusions about the nature of the influence of the separate factors are in complete accord with experimental results with which we are familiar.

We now compare the results of the proposed theory with the convection theory generally accepted at the present time. That theory does not include radiative components and therefore the proposed and accepted theories should be compared in the absence of radiation. In such a case, Eq. (23) takes the form

$$A' = \frac{\lambda}{H}\,\frac{R_1}{PD}\,\frac{T_1 + T_0}{2}. \tag{27}$$

It is easy to see that this equation can be transformed to

$$A' = \frac{a}{D}\,\frac{M_2}{M_1}\cdot\frac{c_p}{H}. \tag{28}$$

According to the convection theory,

$$A_{\text{conv}} = \frac{M_2}{M_1}\,\frac{c_p}{H}. \tag{29}$$

As is clear from a comparison of Eqs. (28) and (29), the convection theory leads to an incorrect expression of the psychrometer coefficient even under conditions where the effect of radiation can be neglected. The factor a/D is approximately 0.84, i.e., is close to one, for a water-vapor and air system. This circumstance is the reason that the convection theory, which is essentially based on an incorrect physical concept, gave approximately correct results when used in meteorological measurements. However, attempts to use it under conditions other than meteorological reveal its inaccuracy at once. Its unsoundness is particularly apparent in applications to systems other than a water-vapor and air system. For example, the error is greater than 100% for humidity measurements in hydrogen. The same error is found in measurements of the partial pressures in air of such materials as xylol, toluol, and chlorbenzol. We compare the results of our proposed theory with the theory of Arnold [9, 10], in which radiative flow of heat is included. We compare only final results without discussing the Arnold theory in detail. According to Arnold, the expression for A tends to a limiting value when $u \to \infty$, which corresponds to the conclusions of the convection theory. As is clear from the preceding, the factor a/D is lost in that case. In the opposite limiting case where $u \to 0$, a value is obtained for A which contains the factor a/D according to Arnold. We recall that according to the proposed theory there should not be any psychrometric difference whatever in stable air (except for the case where the transfer surface has an almost spherical shape).

In conclusion, a few words must be said about existing standard psychrometric tables [11], which were computed under the assumption that the coefficient A was a constant. If one is limited to the operating conditions for a standard aspiration psychrometer, it is impossible to ignore the fact that A changes with temperature in such a case. If one estimates the possible scatter in the values of the coefficient A from 0 to 50°C, one obtains a value of ±15% with respect to the average value. Consequently, the use of such tables (given to three significant figures!) is incorrect.

NOTATION

$j = -\rho D dx/d\tau$	is the diffusional mass flux;
c	is the dimensionless mass concentration;
v	is the velocity;
M_v, M_g	are the molecular masses of vapor and gas, respectively;
x	is the coordinate;
ρ	is the mixture density;
λ	is the thermal conductivity;
c_p	is the isobaric heat capacity;
T	is the temperature;
P	is the total pressure;
p	is the partial pressure of substance evaporating from thermometer surface;
ε	is the Stefan–Boltzmann constant;
σ	is the Stefan–Boltzmann constant;
r	is the radius;
H	is the phase-transition heat;
D	is the diffusion coefficient;
R_c	is the gas constant;
a	is the thermal diffusivity.

Subscripts and Superscripts

1 and 0 refer, respectively, to values in the medium and on the surface of the thermometer bulb.

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